

Four-Generation Low Energy Supersymmetry with a Light Top Quark

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Abstract

A supersymmetric model with four generations is proposed, in which the top quark is approximately degenerate in mass with the W^\pm gauge boson, $m_t \simeq m_W$, leading to values of R_b in better agreement with the present experimental data than in the Standard Model. The model shares many of the good features of the minimal supersymmetric extension of the Standard Model (MSSM), such as the unification of gauge and Yukawa couplings at a common high-energy scale. The model differs from the MSSM by re-interpreting the Tevatron “top-quark” events as the production of the fourth generation quark t' , which decays dominantly to bW^+ . The top quark decays primarily into supersymmetric particles, $t \rightarrow \tilde{t}\tilde{\chi}_1^0$, with $\tilde{t} \rightarrow c\tilde{\chi}_1^0$, thereby evading previous searches. Light supersymmetric particles are predicted to lie in the mass range between 25 and 70 GeV, which together with the fourth generation leptons provide a rich spectrum of new physics which can be probed at LEP-2 and the Tevatron.

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1 Introduction

Recently, the CDF and D0 Collaborations have announced the discovery of the top quark at the Tevatron [1], with a measured mass of $m_t = 176 \pm 8 \pm 10$ GeV and $m_t = 199_{-21}^{+19} \pm 22$ GeV, respectively. Both measurements are in excellent agreement with the top quark mass deduced by the LEP global analysis of precision electroweak measurements. The most recent numbers quoted by the LEP Electroweak Working Group from a Standard Model fit with a Higgs mass of 300 GeV are: $m_t = 170 \pm 10_{-19}^{+17}$ GeV [$180_{-9}^{+8} {}_{-20}^{+17}$ GeV] based on the global average of the data from the four LEP detectors and excluding [including] data from the SLD Collaboration [2], where the last positive and negative error quoted corresponds to alternative Higgs mass choices of 1 TeV and 60 GeV, respectively. The LEP determination of m_t derives from the sensitivity of electroweak observables in Z decays to virtual top quark exchange, which enters in two distinct ways. First, top quark loops in gauge boson self-energies (the so-called oblique corrections) can directly affect the properties of the Z and W^\pm . The most important oblique correction is the top-quark contribution to the electroweak ρ parameter [3], which is given by $\rho = 1 + \Delta\rho$, where $\Delta\rho \simeq 3G_F m_t^2 / 8\pi^2 \sqrt{2}$. Second, top-quark loops can contribute to three-point (and higher) interactions (these are the non-oblique corrections). The most significant non-oblique correction to Z decays arises in the vertex correction to $Z \rightarrow b\bar{b}$, which is also quadratically sensitive to the top quark mass. The size of the oblique corrections, which can be inferred from the present precision data analysis, is in very good agreement with the Standard Model theoretical expectations. In contrast, the measured value of the rate for $R_b \equiv \Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{hadrons})$ lies more than three standard deviations above the Standard Model prediction.

If the deviation in the observed value of R_b holds under further experimental scrutiny, then this would be a signal of new physics beyond the Standard Model, arising (presumably) from the virtual exchange of new particles. In the literature, models of low-energy supersymmetry have been examined in which R_b is slightly enhanced above the Standard Model prediction [4,5,6,7,8,9,10,11,12,13]. In such models, the global fit to the electroweak data is improved. Note that in order to improve the Standard Model fit, one must approximately maintain the size of the Standard Model oblique corrections while modifying the $Zb\bar{b}$ interaction. In the minimal supersymmetric extension of the Standard Model (MSSM), this is possible if one takes large [small] values of the mass parameters of the scalar super-partners of the left [right] handed top quark, and small values of the Higgs superfield mass parameter μ .

In this paper, we take an alternative approach. If one considers the R_b measurement alone within the context of the Standard Model, one would conclude that $m_t \lesssim m_W$. To avoid conflict with the measurement of m_t at the Tevatron (by direct means) and at LEP (by indirect means) as described above, one must invoke new physics to explain the CDF and D0 “top-quark” events and new virtual effects to generate the oblique radiative corrections observed in precision electroweak experiments. In addition, one must explain why such a light top-quark has escaped detection in previous experiments at hadron colliders. Surprisingly, a model with such a light top-quark exists which is not in conflict with present experimental observations. The model consists of the

MSSM extended to four generations, with a light top-squark such that $t \rightarrow \tilde{t}\tilde{\chi}_1^0$ with nearly 100% branching ratio (where $\tilde{\chi}_1^0$ is the lightest neutralino, assumed to be the lightest supersymmetric particle). The CDF and D0 “top-quark” events are identified as $t' \rightarrow bW^+$, and the oblique radiative corrections observed in precision electroweak data are approximately saturated by loops of third and fourth generation quarks and squarks.

In section 2 we review in more detail the top-quark mass dependence of precision electroweak observables, and discuss the implications of the recent measurement of R_b (and the corresponding measurement of R_c). We discuss some of the requirements for the viability of the light top quark explanation of the observed discrepancy in R_b . In section 3 we analyze the restrictions on the fourth generation masses due to a variety of phenomenological and theoretical constraints. By imposing the requirement that none of the couplings of the model become nonperturbative below the grand unification (GUT) scale, we find that all fourth generation fermion–Higgs Yukawa couplings approximately unify at the GUT scale while approaching quasi-infrared fixed point values at the electroweak scale. In sections 4–6 we provide a detailed phenomenological profile of our model, and discuss the constraints on its parameters, with particular attention given to the implications of the precision electroweak data and flavor changing processes. Section 7 summarizes the successes and the shortcomings of our model. Some technical details are included in the Appendix. A preliminary version of this work was presented in Ref. [14].

2 The top quark mass from precision electroweak data

Experiments at LEP and SLC measure more than fifteen separate electroweak observables in Z decay events. Global fits of these observables exhibit a remarkable consistency with Standard Model expectations, with two notable exceptions. Defining $R_q \equiv \Gamma(Z \rightarrow q\bar{q})/\Gamma(Z \rightarrow \text{hadrons})$, with $q = b, c$, the LEP Electroweak Working Group global fit yields [2]

$$R_b = \begin{cases} 0.2219 \pm 0.0017, & \text{LEP/SLC global fit;} \\ 0.2156, & \text{Standard Model prediction,} \end{cases} \quad (2.1)$$

which is a 3.7σ discrepancy, and

$$R_c = \begin{cases} 0.1543 \pm 0.0074, & \text{LEP/SLC global fit;} \\ 0.1724, & \text{Standard Model prediction,} \end{cases} \quad (2.2)$$

which is a 2.5σ discrepancy. One other LEP measurement relevant to this discussion is the $\alpha_s(m_Z)$ determination [9,15,16,17] from the total hadronic width of the Z . Based on the measurement of $R_\ell \equiv \Gamma_{\text{had}}/\Gamma_{\ell\ell}$, the Standard Model fit yields [2] $\alpha_s(m_Z) = 0.126 \pm 0.005 \pm 0.002$ (where the last error quoted corresponds to varying the Higgs mass from 60 GeV to 1 TeV). It is interesting to note that the LEP determination of $\alpha_s(m_Z)$ tends to be somewhat higher than the extrapolated value of $\alpha_s(m_Z)$ obtained from lower energy measurements.

One must be very careful in interpreting the R_b and R_c discrepancies from Standard Model expectations. The experimental procedures that identify b and c quarks in Z

decays are difficult and prone to large systematic errors. Regarding the R_c measurement, we note first that the quoted error is larger, and the statistical significance of the deviation from the Standard Model prediction is smaller, than those of R_b . Second, it is very difficult to imagine new physics processes that could produce such a large deviation in R_c . To understand the latter point, suppose new physics were responsible for both the R_b and R_c discrepancies. Because $R_b + R_c$ is less than the Standard Model prediction, one is led to a distressing conclusion. Since $\alpha_s(m_Z)$ is determined from the experimentally measured Γ_{had} , a negative contribution of new physics to this quantity implies that the perturbative QCD corrections to Γ_{had} should be larger. That is, $\alpha_s(m_Z)$ determined from Γ_{had} would be larger than the number quoted above, in significant conflict with the low-energy determinations. This argument implicitly assumes that there are no new physics contributions to R_q where q is a light quark. On the other hand, there is no known source of new physics that can modify R_q sufficiently to compensate the deficit in $R_b + R_c$ to avoid the above conclusion. Thus, we are inclined to discount the measured value of R_c above, and assume that its true value is close to the Standard Model expectation. Should we discount the measured value of R_b as well? Further experimental analysis is required to clarify the situation. Here, we simply note that if new physics contributes to R_b but not to R_c and R_q , then because R_b is above the Standard Model prediction, $\alpha_s(m_Z)$ determined from Γ_{had} will be *lower*, potentially closer to the low energy α_s determinations [9,15,16,17].

Henceforth, we assume that R_c is given by its Standard Model prediction. In the experimental determination of R_b , there is some contamination of $c\bar{c}$ events in the $b\bar{b}$ sample that must be subtracted. This subtraction depends on the value of R_c assumed. Fixing R_c to its Standard Model value, a slightly smaller value of R_b is found by the Electroweak working group compared to the value quoted above:

$$R_b = 0.2205 \pm 0.0016, \quad \text{LEP/SLC global fit}, \quad (2.3)$$

roughly a three standard deviation discrepancy from the Standard Model prediction.

The $Zb\bar{b}$ vertex corrections can also significantly affect the left-right $b\bar{b}$ asymmetry \mathcal{A}_b ,

$$\mathcal{A}_b = \frac{g_L^2 - g_R^2}{g_L^2 + g_R^2}, \quad (2.4)$$

where g_L (g_R) are the couplings of the left (right) handed bottom quarks to the Z . The corrections to R_b and \mathcal{A}_b can be parameterized as a function of the corrections to the left- and right-handed bottom quark vertices,

$$\frac{\delta \mathcal{A}_b}{\mathcal{A}_b} = \frac{4f_R f_L}{f_L^4 - f_R^4} [f_R \delta g_L - f_L \delta g_R], \quad (2.5)$$

$$\frac{\delta R_b}{R_b} = \frac{2(1 - R_b)}{f_L^2 + f_R^2} [f_R \delta g_R + f_L \delta g_L], \quad (2.6)$$

where $f_L = 1/2 - \sin^2 \theta_W(m_Z)/3$ and $f_R = -\sin^2 \theta_W(m_Z)/3$ are the tree level couplings of the left and right handed bottom quarks to the Z . The dominant, top quark mass

dependent, one loop $Zb\bar{b}$ vertex corrections affect only the left-handed bottom quark coupling to the Z ,

$$\delta g_L = -\frac{\alpha}{16\pi \sin^2 \theta_W} m_t^2. \quad (2.7)$$

The large difference between the values of f_L and f_R implies that for $\delta g_R = 0$, $\delta R_b/R_b \simeq 11.5 \delta \mathcal{A}_b/\mathcal{A}_b$. Moreover, the current determination of \mathcal{A}_b at SLC is still subject to large experimental errors [2]

$$\mathcal{A}_b = \begin{cases} 0.841 \pm 0.053 & \text{LEP/SLC global fit;} \\ 0.935, & \text{Standard Model prediction.} \end{cases} \quad (2.8)$$

Therefore, \mathcal{A}_b does not provide at present any significant constraint on the value of the top quark mass.

Barring additional experimental problems, and even after taking into account the possible systematic errors due to an incorrect charm background subtraction, the discrepancy in R_b quoted in Eq. (2.3) is large enough as to raise concerns about the Standard Model interpretation of the data. As noticed above, if one extracts the prediction of the top quark mass from the R_b measurement alone, one would conclude that the top quark is much lighter than the value inferred from the global LEP fit to the data or from the direct Tevatron measurements. Of course, with such a light top quark, one must address the obvious questions of why such a top quark has not yet been discovered, and what is the particle recently announced by the CDF and D0 Collaborations that is observed to decay into bW^+ .

If the top quark were sufficiently light, then $W^+ \rightarrow t\bar{b}$ would be kinematically allowed; this would modify the total width of the W^\pm . But Γ_W can be measured at hadron colliders indirectly by studying the ratio of production cross section times leptonic branching ratio of the W^\pm and Z . The most recent analysis of this kind, reported by the CDF collaboration [18], finds $m_t > 62$ GeV at 95% CL. (The most recent D0 data [19] implies a similar bound. A global average of all hadron collider data would raise this limit by a few GeV.) Direct searches for the top quark at hadron colliders assume that an observable fraction of top quark decays results in a lepton in the final state. For example, the D0 collaboration ruled out the mass range $m_W + m_b \lesssim m_t < 131$ GeV, assuming that the decay $t \rightarrow bW^+$ is not unexpectedly suppressed [20]. Previous top quark searches at hadron colliders were able to close the window between 62 GeV and $m_W + m_b \simeq 85$ GeV, assuming that $t \rightarrow bW^*$ is the dominant top-quark decay mode. However, in the latter case, the final state is three-body since W^* is virtual. If the top quark possessed any two-body decay modes (due to new physics processes), and if these modes rarely produced leptons, then a top quark in this mass region could have escaped detection in all previous top quark searches.

One possible scenario leading to two-body decay modes of the top quark arises if one assumes the existence of a light charged Higgs particle in the spectrum, $m_{H^\pm} < m_W$. Recent results from the CDF Collaboration rule out a substantial portion of the allowed parameter space [21] based on a direct search for $t \rightarrow bH^+$. Additional constraints arise from considering the contribution of virtual charged Higgs effects to heavy quark decays. In particular, one-loop charged Higgs exchange can significantly modify the predictions for rare bottom quark decay rates. Indeed, from the requirement

of obtaining an acceptable rate for the branching ratio $\text{BR}(b \rightarrow s\gamma)$, one can show that this scenario is ruled out [22], unless the enhancement of this branching ratio due to the presence of a light charged Higgs boson is compensated by the contribution of new particles appearing at the electroweak scale.

Thus, we turn to an alternative scenario, based on low-energy supersymmetry with a light top-squark, in which the decay $t \rightarrow \tilde{t}\tilde{\chi}_1^0$ is kinematically allowed (where \tilde{t} is the lightest top squark and $\tilde{\chi}_1^0$ is the lightest neutralino) [23]. Experimental searches for both \tilde{t} and $\tilde{\chi}_1^0$ place constraints on their masses, but do not rule out the possibility of $M_{\tilde{t}} + M_{\tilde{\chi}_1^0} < m_W$. In particular, the LEP neutralino and chargino searches [24] imply a limit on the lightest neutralino mass which typically lies between 20 and 25 GeV (although lighter neutralino masses cannot be ruled out if $\tan\beta$ is very close to one). Using this result and the limits on the top squark mass from searches at LEP and at the Tevatron [25], one finds that the mass region $50 \text{ GeV} \lesssim M_{\tilde{t}} \lesssim 60 \text{ GeV}$ (with $M_{\tilde{\chi}_1^0} \gtrsim 20 \text{ GeV}$) cannot be excluded.¹

To be definite, we choose $m_t \simeq m_W$, $M_{\tilde{t}} \simeq 50 \text{ GeV}$ and $M_{\tilde{\chi}_1^0} \simeq 25 \text{ GeV}$. Then, the dominant decay chain is $t \rightarrow \tilde{t}\tilde{\chi}_1^0$ followed by $\tilde{t} \rightarrow c\tilde{\chi}_1^0$ through a one-loop process [23], which rarely produces a hard isolated lepton. Hence, these events would not have been detected at hadron colliders. But, now we must reconsider the recent CDF and D0 discoveries and the LEP “measurement” of m_t . As anticipated in section 1, we propose to account for these results by introducing a fourth generation of quarks (and leptons) plus their supersymmetric partners. Then, $t' \rightarrow bW^+$ can be the source of the CDF and D0 events, while the effects of the third and fourth generation quarks and squarks contributing to the oblique corrections are large enough to be consistent with LEP precision electroweak data.

3 Fourth generation fermion masses and unification

For the t' to be consistent with the CDF and D0 “top-quark” events, its mass should be close to the world average, $m_{t'} = 180 \pm 12 \text{ GeV}$. We assume that the t' mass is given by $m_{t'} \simeq 175 \text{ GeV}$. Moreover, in order to produce the observed Tevatron signal, the dominant t' decay must be $t' \rightarrow bW^+$, in which the generation number changes by one unit. This means that two body generation number conserving decays, such as $t' \rightarrow b'W^+$ or the supersymmetric decay modes $t' \rightarrow \tilde{b}'\tilde{\chi}^\pm$ and $t' \rightarrow \tilde{t}'\tilde{\chi}^0$, must be kinematically forbidden. Of course, three-body generation number conserving decays will be possible (*e.g.*, $t' \rightarrow b'W^*$, where the virtual W^* decays in the usual way to a fermion antifermion pair). It follows that the t' - b mixing angle ($V_{t'b}$) must not be too small; otherwise the three-body generation conserving decays will dominate. For example, we find:

$$\frac{\Gamma(t' \rightarrow b'W^*)}{\Gamma(t' \rightarrow bW)} = \frac{9G_F m_{t'}^2 |V_{t'b'}|^2}{\pi^2 \sqrt{2} |V_{t'b}|^2} \int_0^{1-2\sqrt{x+x}} \frac{z(1-z+x)\sqrt{(1-z+x)^2-4x}}{(1-zm_{t'}^2/m_W^2)^2} dz, \quad (3.1)$$

¹The most recent LEP run, at $\sqrt{s} = 136 \text{ GeV}$, is not sensitive to this range of masses in the case of maximal top squark mixing (the latter is a necessary feature of our model).

where $x \equiv m_{b'}^2/m_{t'}^2$. Since the rate of the CDF and D0 “top-quark” events is consistent with the QCD prediction for $t\bar{t}$ production under the assumption that $\text{BR}(t \rightarrow bW^+) \simeq 1$, a reinterpretation of these events as $t'\bar{t}'$ production (followed by $t' \rightarrow bW^+$) requires $\text{BR}(t' \rightarrow bW^+)$ to be near 1. At present, there are no relevant constraints on the mixing parameter $V_{t'b}$. We assume that $V_{t'b}$ lies between $V_{cb} = 0.04$ and $V_{us} = 0.2$, although even larger values may be allowed in some circumstances [26]. For definiteness, we choose $|V_{t'b}/V_{t'b'}| = 0.1$.

In Figure 1, we show the dependence of the branching ratio $\text{BR}(t' \rightarrow bW^+)$ on the b' mass. We see that there is a sharp increase of this branching ratio as soon as $m_{b'} + m_W \gtrsim m_{t'}$. Then, if we require $\text{BR}(t' \rightarrow bW^+) \gtrsim 0.75$ for the chosen value of $V_{t'b}$, it follows that $m_{b'} \gtrsim 105$ GeV. Such a large value of the fourth generation down-type quark mass implies that the custodial SU(2) symmetry breaking is softer than in the three generation Standard Model case. As we shall discuss in section 5, this has important consequences for the value of the ρ -parameter. In particular, values of the b' mass too close to 175 GeV would immediately imply a suppression of the one-loop contribution to the ρ -parameter, which is in conflict with the present precision electroweak data. Hence, the b' mass should either be larger than the t' mass or close to its experimental lower bound, $m_{b'} \simeq 105$ GeV. Having introduced a fourth generation of quarks, the simplest way to avoid electroweak gauge anomalies is to introduce a fourth generation of sequential leptons. In order to be consistent with present LEP-1 bounds, their masses must be larger than 45 GeV (see discussion below).

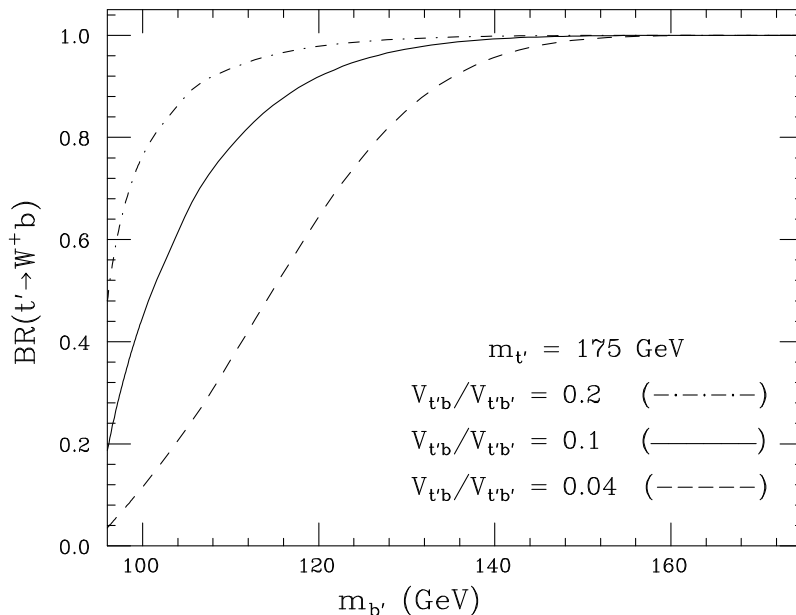


Figure 1. Branching fraction $\text{BR}(t' \rightarrow bW^+)/\text{BR}(t' \rightarrow b'W^+)$ as a function of the fourth generation bottom mass $m_{b'}$.

In the absence of any assumption about the nature of the physics at high energy scales, it is simple to construct a four-generation model, which fulfills all the relevant experimental bounds, by simply pushing up the b' quark mass above the t' quark mass. In contrast, we find that our four-generation model is remarkably well-constrained if we

demand that none of the gauge or Yukawa couplings blow up below the GUT scale (or, in the absence of grand unification, the Planck scale). In the latter case, the parameters of the model are almost uniquely determined, and the model is on the verge of being ruled out (or poised for experimental discovery).

One of the strongest motivations for low-energy supersymmetric extensions of the Standard Model comes from the fact that such models can be realized as effective low energy descriptions of theories in which all known low-energy gauge interactions unify at a high energy scale. In the MSSM, the values of the low energy gauge couplings and third generation fermion masses are consistent with the unification of gauge and Yukawa couplings at a single high energy scale $M_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV [27]. A fourth generation of fermions and their supersymmetric partners can be accommodated in complete SU(5) superfield multiplets, $\mathbf{10} + \bar{\mathbf{5}}$, and does not affect the successful $\sin^2 \theta_W(m_Z)$ prediction coming from the gauge coupling unification. The two-loop running of the gauge couplings is only slightly modified by the presence of the fourth generation, leading to a value of $\alpha(M_{\text{GUT}}) \simeq 0.08$ for the unified gauge coupling and a slightly larger value of the unification scale, $M_{\text{GUT}} \simeq 5 \times 10^{16}$ GeV. Moreover, taking $\alpha_{em}(m_Z)$ and $\sin^2 \theta_W(m_Z)$ as input parameters, the two loop corrections enhance slightly the predicted values of $\alpha_s(m_Z)$ with respect to the value obtained in the MSSM [28]. This enhancement is partially suppressed by the effect of the large fourth generation Yukawa couplings, which happen to be close to their fixed point values. In any case, as in the MSSM, the predicted value of the strong gauge coupling, $\alpha_s(M_Z) \simeq 0.13$, is somewhat high to be consistent with the values of the strong gauge coupling extracted from LEP within the assumptions of this model. However, it is enough to assume the presence of small threshold corrections at the grand unification scale to produce a predicted value of $\alpha_s(M_Z)$ that is consistent with the present experimental data.

Consider next the requirement of perturbative consistency of the Yukawa sector of the theory up to scales of order M_{GUT} . The low energy Higgs- b' quark Yukawa coupling is related to the running mass $m_{b'}$ by

$$h_{b'} = \frac{\sqrt{2}m_{b'}}{v \cos \beta}, \quad (3.2)$$

where $v = 246$ GeV and $\tan \beta$ is the ratio of the two Higgs vacuum expectation values. For a fixed value of $m_{b'}$ an upper bound on $\tan \beta$ may be hence obtained. For a physical mass $m_{b'} \gtrsim 105$ GeV, only small values of $\tan \beta < 2$ are allowed. On the other hand, the Higgs- t' quark Yukawa coupling is related to its running mass by

$$h_{t'} = \frac{\sqrt{2}m_{t'}}{v \sin \beta}. \quad (3.3)$$

Consequently, low values of $\tan \beta$ tend to push the t' mass to relatively low values, unless its Yukawa coupling is itself close to its perturbativity bound, corresponding to the quasi-infrared fixed point value of this quantity. Large values of $h_{t'}$ have the additional effect of changing the renormalization group evolution of $h_{b'}$, leading to lower values of $m_{b'}$ (or equivalently, lower values of $\tan \beta$ at fixed $m_{b'}$). As a result, in order to obtain values of the fourth generation b' and t' masses consistent with the limits

quoted earlier, $m_{t'} \simeq 175$ GeV, $m_{b'} \gtrsim 105$ GeV, two properties must be fulfilled:

1. The b' mass must be close to its lower bound. Remarkably, this conclusion coincides with the one obtained from the considerations of the fourth generation contributions to the ρ -parameter, discussed in more detail in section 5.
2. The b' and t' Yukawa couplings must lie close to their quasi infrared fixed point values [29].

Therefore, the b' mass is essentially fixed by perturbativity arguments. Moreover, since we are close to the infrared fixed point values of the Yukawa couplings, $\tan \beta$ is also fixed. Figure 2 shows the resulting fourth generation quark masses as a function of $\tan \beta$ assuming that both Yukawa couplings lie close to their fixed point values. For $m_{t'} \simeq 175$ GeV and $m_{b'} = 105$ GeV; we obtain $\tan \beta \simeq 1.6$. Our picture is consistent with the unification of the t' and b' Yukawa couplings at M_{GUT} and, unlike the case of gauge coupling unification, the predicted low energy values are highly insensitive to possible moderate threshold corrections at M_{GUT} .

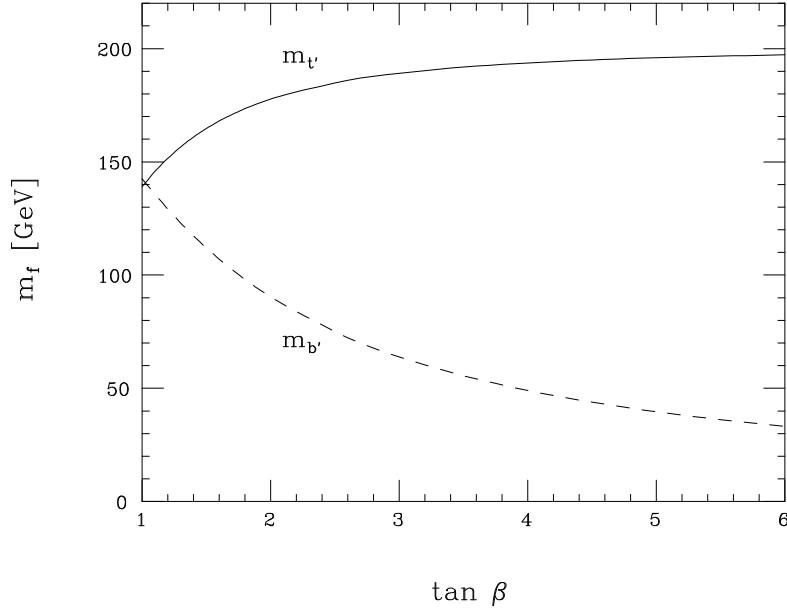


Figure 2. Plot of the b' and t' quark masses as a function of $\tan \beta$ assuming that both lie close to their fixed point values.

The Tevatron may be able to rule out the existence of the b' with mass $m_{b'} \simeq 105$ GeV (although the existing experimental limits only rule out b' quarks of mass up to 85 GeV [24]). If kinematically allowed, the decay $b' \rightarrow \tilde{t}\tilde{\chi}_1^-$ would be the dominant decay mode. If disallowed, there would be a competition between $b' \rightarrow Wc$ (a change of two generations) and $b' \rightarrow W^*t$ (a change of one generation, but suppressed by three-body phase space). One option is to choose $|V_{b'c}| \ll |V_{t'b}|$ to remove the possibility of $b' \rightarrow Wc$. Then, all b' decays would result in the final state $W^*c\tilde{\chi}_1^0\tilde{\chi}_1^0$. There are no published limits that exclude such a b' . However, a dedicated search at the Tevatron

should be able to discover or exclude such events.

The fourth generation lepton Yukawa couplings are also strongly constrained by the experimental bound on the lepton masses $m_{\ell'} > 45$ GeV. When applied to the neutrino mass, this bound implies that the right-handed Majorana mass, if present, must be of order of the electroweak scale. In addition, the bounds on the τ' lepton mass imply that its Yukawa coupling must be larger than $\mathcal{O}(1)$ at the grand unification scale. We find that the infrared fixed point value for $m_{\tau'}$ is very close to the current experimental bound on this quantity. Although there is more freedom in the $\tau'-N_{\tau'}$ sector, one can also add the requirement that both fourth generation Yukawa couplings take large values at M_{GUT} (in order to maximize their masses). Figure 3 shows the predicted values for the fourth generation lepton masses assuming that the neutrino $N_{\tau'}$ is a Dirac neutrino, and that the four fourth-generation Yukawa couplings are close to their fixed point values. Then, for $\tan \beta \simeq 1.6$, the resulting lepton masses are: $m_{\tau'} \simeq 50$ GeV and $m_{N_{\tau'}} \simeq 80$ GeV. It is interesting to note that these lepton masses are consistent with the unification of all *four* fermion-Higgs Yukawa couplings at the GUT scale. Moreover, these masses lie above the corresponding bounds from LEP-1, but both are in the kinematic reach of LEP-2.

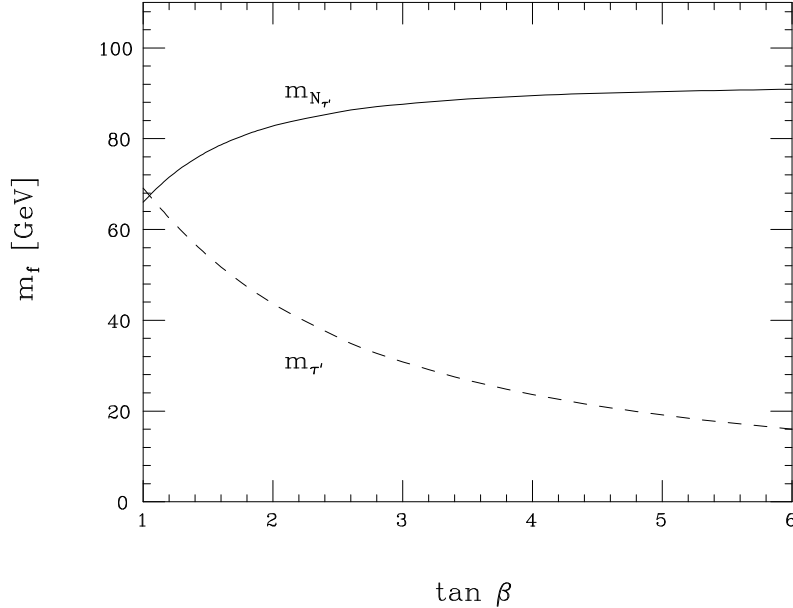


Figure 3. The same as Fig. 2 but for the τ and the fourth generation neutrino masses.

In general, the fourth generation neutrino mass matrix can contain both an off-diagonal Dirac mass, m_D and a right-handed diagonal Majorana mass, m_M . Thus, in the more general case, the two fourth generation neutrino eigenstates are split by the see-saw mechanism. The lighter state has mass given by

$$m_{N_{\tau'}} = \frac{1}{2} \left[\sqrt{m_M^2 + 4m_D^2} - m_M \right]. \quad (3.4)$$

The infrared fixed point limit of m_D coincides with that of $m_{N_{\tau'}}$, shown in Figure 3. There is no corresponding limit on the Majorana mass m_M . For $m_D \simeq 80$ GeV, it

is easy to find the value of m_M (roughly equal to m_D) such that the light neutrino state becomes approximately degenerate with the τ' . This observation implies that a range of parameters exists where $N_{\tau'}$ is a few GeV lighter than the τ' . Due to the near degeneracy of the two states, it would be extremely difficult in this case to detect the τ' at LEP-2 through its dominant decay $\tau' \rightarrow N_{\tau'} W^*$, unless the $N_{\tau'}$ decayed inside the collider detector.

4 Phenomenological Restrictions and Implications

In section 2, we noted that our model must possess a rather light top-squark (stop) whose mass is about 50 GeV. In order to achieve such a small $M_{\tilde{t}}$, there must be substantial \tilde{t}_L - \tilde{t}_R mixing. $M_{\tilde{t}}^2$ is given by the smallest eigenvalue of the stop squared-mass matrix

$$\begin{pmatrix} M_Q^2 + m_t^2 + c_L m_Z^2 & m_t(A_t - \mu \cot \beta) \\ m_t(A_t - \mu \cot \beta) & M_U^2 + m_t^2 + c_R m_Z^2 \end{pmatrix}, \quad (4.1)$$

where M_Q , M_U , and A_t are soft-supersymmetry-breaking parameters, $c_L \equiv (\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W) \cos 2\beta$, $c_R \equiv \frac{2}{3} \sin^2 \theta_W \cos 2\beta$, and μ is the supersymmetric Higgs mass parameter. A light stop implies that the off-diagonal terms in Eq. (4.1) are of the same order as the geometric average of the diagonal terms,

$$(m_Q^2 + m_t^2)(m_U^2 + m_t^2) \simeq m_t^2 X_t^2 \quad (4.2)$$

where $X_t \equiv A_t - \mu \cot \beta$. This can be accomplished either by taking X_t rather large or by pushing one of the soft supersymmetry breaking mass parameters to low values. However, the latter possibility is severely constrained by the constraints on the ρ -parameter and the bottom squark mass, as discussed in section 5.

If there is large mixing in the third generation squark sector, one would naively expect that the fourth generation squark mixing would be large as well. If $X_{t'}$ were of order X_t , the mixing in the fourth generation squark sector would be too large, driving the smallest eigenvalue of the \tilde{t}'_L - \tilde{t}'_R squared-mass matrix negative. Remarkably, due to the infrared fixed-point behavior of the fourth generation, $A_{t'}$ is driven to a value that is independent of its high energy value [30]. Roughly, $A_{t'} \simeq -1.5m_{1/2}$ where $m_{1/2}$ is the high-energy (GUT-scale) value of the gaugino Majorana mass. In contrast, due to the smaller values of the top quark Yukawa coupling, the top squark mixing parameter A_t is strongly dependent on its high energy value and hence can be taken as an independent parameter. Moreover, choosing μ negative and A_t positive enhances the third generation squark mixing while somewhat suppressing the fourth generation squark mixing.

The constraints on the chargino and neutralino sectors provide information about the supersymmetric mass parameter μ . If the gaugino Majorana mass parameters are unified with a common GUT-scale mass given by $m_{1/2}$, then the gluino, chargino and neutralino masses are determined by $m_{1/2}$, μ , and $\tan \beta$. This is the situation in most grand unified theories (assuming that the threshold corrections, proportional to the GUT-symmetry breaking order parameter, are small). In order to obtain the desired

hierarchy between the lightest chargino and neutralino masses, our model prefers the region of parameter space where $m_{1/2} \ll |\mu|$ (with μ negative). Then, our choice of $M_{\tilde{\chi}_1^0} \simeq 25$ GeV fixes

$$m_{1/2} \simeq \frac{\alpha(M_{\text{GUT}})}{\alpha_2(M_Z)} M_2 \simeq 110 \text{ GeV}. \quad (4.3)$$

In general, small threshold corrections at the grand unification scale will modify the exact relation among the gaugino masses. Assuming corrections of at most 25% in these relations, the typical values for the masses of the other light chargino and neutralino states are $M_{\tilde{\chi}_1^\pm} \simeq M_{\tilde{\chi}_2^0} \simeq 55\text{--}70$ GeV. The dominant decay of the chargino would be $\tilde{\chi}^+ \rightarrow \tilde{t} \bar{b}$, followed by $\tilde{t} \rightarrow c \tilde{\chi}^0$. Note that the signature for this decay chain consists of bottom and charm quark initiated jets plus missing energy. In particular, such events would be phenomenologically distinct from the usual three body decays $\tilde{\chi}^+ \rightarrow \tilde{\chi}^0 W^+$ expected in the MSSM for a light gaugino-like chargino. The most recent data from LEP at $\sqrt{s} = 136$ GeV places severe constraints on the chargino parameters of our model, but does not yet rule out the entire parameter space. The choice of $m_{1/2}$ also fixes the gluino mass; we find $M_{\tilde{g}} \simeq 150\text{--}200$ GeV. The dominant decay of this gluino would be $\tilde{g} \rightarrow \tilde{t} \bar{t}$ (or its charge-conjugated state) followed by $\tilde{t} \rightarrow c \tilde{\chi}_1^0$. Such a gluino cannot be ruled out by present Tevatron limits.

The mass of the lightest CP-even Higgs boson should lie above the LEP lower limit. For $\tan\beta = 1.6$, the tree-level *upper bound* on the light Higgs mass is $m_{h^0} \leq m_Z |\cos 2\beta| \simeq 40$ GeV, which would have been detected at LEP. However, radiative corrections can substantially raise the tree-level upper bound [31]. In our model, the most important radiative corrections are associated with the fourth generation squark and quark masses. We employ the radiative corrections to the Higgs masses as outlined in Refs. [32,33,34,35,36]; it is crucial to include the complete fourth generation squark threshold effects, which depend on the off-diagonal squark mixing terms. For the values of $A_{t'}$ and $A_{b'}$ obtained from the infrared fixed point constraints and taking the μ parameter negative and of order the soft-supersymmetry-breaking diagonal squark mass parameters, we find that the experimental Higgs mass bound is fulfilled even for relatively low values of the fourth generation squark mass parameters, $m_{Q'} \gtrsim 200$ GeV. For example, for $m_{Q'} \simeq 250$ GeV, we find $m_{h^0} \simeq 65\text{--}70$ GeV as a typical range of values for the light CP-even Higgs mass, which is just above the present LEP limits.

5 Precision Measurements and the Top Quark Mass

Since the fourth generation quarks are relatively heavy and presumably are weakly mixed with the light quarks, their contributions to electroweak observables through virtual exchange occurs dominantly through loop corrections to the gauge boson self-energies (*i.e.*; the oblique corrections). In contrast, the third generation quark virtual effects are still responsible for the radiative corrections to $Z \rightarrow b \bar{b}$ and $b \rightarrow s \gamma$. However, the size of the virtual effects cannot be evaluated before specifying the value of the supersymmetric Higgs mass parameter μ .

The μ parameter is constrained by the oblique corrections to the ρ -parameter. Let $\Delta\rho$ be the one-loop contribution to ρ arising from the various sectors of the supersymmetric model. Because the top quark mass is less than half of its standard value, the contribution of the t - b doublet to $\Delta\rho$ is reduced by a factor of four. Since the fourth generation top quark mass $m_{t'}$ is roughly of the same order as the standard top one, one might expect that the fourth generation fermions could make up the deficit in $\Delta\rho$. However, this is not the case due to the large value of $m_{b'}$. Using

$$\Delta\rho(t, b) = \frac{g^2 N_c}{64\pi^2 m_W^2} \left[m_t^2 + m_b^2 - \frac{2m_t^2 m_b^2}{m_t^2 - m_b^2} \log \left(\frac{m_t^2}{m_b^2} \right) \right], \quad (5.1)$$

where $N_c = 3$ is the color factor, it follows that the fourth generation fermions contribution to $\Delta\rho$ (for $m_{t'} = 175$ GeV and $m_{b'} = 105$ GeV) is approximately the same as that from the third generation. Hence, the supersymmetric particle contributions must account for roughly a half of the total value of $\Delta\rho$. (Explicit formulae can be found in Appendix A.) This requirement places severe restrictions on the third and fourth generation squark mass parameters M_Q , M_U and M_D [see Eq. (4.1)]. One must maximize the off-diagonal squark mixing while keeping the diagonal squark mass parameters as small as possible. However, as discussed in section 4, the latter cannot be too small; otherwise the radiative corrections to the light Higgs mass will be reduced leading to a value of m_{h^0} below the current LEP bound.

To obtain a sufficient squark contribution to $\Delta\rho$ is not a simple task. The squark contribution to $\Delta\rho$ arises only from loops involving \tilde{t}_L and \tilde{b}_L . Since $m_b \ll m_Z$, there is very little mixing in the bottom squark sector, and it follows that $M_{\tilde{b}_L} \simeq M_Q$ should be larger than about 150 GeV. Furthermore, a large value of M_Q implies that a significant squark contribution to $\Delta\rho$ may only come from large values of the off-diagonal squark mixing. Indeed, taking into account the requirement that $M_{\tilde{t}} \simeq 50$ GeV, the breakdown of the custodial $SU(2)$ symmetry is best achieved by taking: (i) large values of the right handed squark mass parameter ($M_U \gg M_Q$) to maximize the left handed component of the lightest stops; (ii) moderate or large values of M_Q to maximize the mass difference between the left handed sbottom and the lightest stop; and (iii) large values of the off-diagonal stop mixing [as required by Eq. (4.2)]. The breakdown of the custodial symmetry arises predominantly as a result of the large squark mixing effects. By this procedure, we can make the stop contribution to the ρ parameter as large as desired. However, the large hierarchy between the parameters M_U and M_Q and the large values of the mixing parameter X_t needed to reproduce the observed value of the ρ -parameter seems rather ad-hoc. Thus, in what follows, we focus on a more natural choice of parameters where M_U is of order M_Q , although large values of X_t are required in this case as well.

There is a danger that large values of the mixing parameter X_t may render the Standard Model vacuum state (globally) unstable [37,11,38]. In this case the true vacuum would break electric or color charge due to a non-zero vacuum expectation value of some squark or slepton field. However, the requirement of global stability of the electric and color charge conserving vacuum is perhaps too strong. The Standard Model vacuum can be metastable as long as its lifetime is sufficiently long relative to the age of the universe. The weaker condition of vacuum metastability can permit the

large values of X_t required by our model [39].

The $\Delta\rho$ constraint essentially fixes the allowed range of soft-supersymmetry breaking diagonal squark masses. Consider first the third generation parameters. We take $M_Q \simeq M_U$, where the squark mixing parameter X_t is chosen in such a way that the light stop has a mass fixed at 50 GeV. In this case, the contribution of the third generation squarks to $\Delta\rho$ is too small unless M_Q lies *above* 200 GeV. The larger we take M_Q , the larger we must tune A_t in order to preserve the light stop. This in turn raises $\Delta\rho$. For example, values of $m_Q \simeq 300$ GeV can generate a contribution from the third generation squarks which is of the same order as $\Delta\rho(t, b)$. However, in this case, one must also tolerate a rather large value of $X_t \simeq 1$ TeV.

Non-negligible mixing in the fourth generation also enhances the fourth generation squark contributions to $\Delta\rho$. The maximum effect is limited by the lower bound on the mass of \tilde{b}' . In order that $t' \rightarrow bW^+$ remain the dominant t' decay, one must kinematically forbid $t' \rightarrow \tilde{b}'\tilde{\chi}_1^+$. Given $M_{\tilde{\chi}_1^\pm} \simeq 55\text{--}70$ GeV, a value of $M_{\tilde{b}'} \simeq 120$ GeV is a comfortable choice. Since $A_{t'}$ and $A_{b'}$ are determined by their infrared fixed point behavior, for a given value of the left and right handed squark mass parameters, the value of μ which maximizes the fourth generation squark contribution to $\Delta\rho$ can be obtained. All the phenomenological constraints have now forced the parameters of the model into a very narrow corner of parameter space.

It is convenient to parameterize the oblique radiative corrections in terms of the Peskin-Takeuchi variables [40] S , T and U . Here $T \equiv \alpha^{-1}\Delta\rho$ (where $\alpha^{-1} \simeq 137$) is the most sensitive variable (although some interesting restrictions can be obtained by considering S). Langacker has performed a global analysis of precision electroweak data [41], assuming that $m_t = 80$ GeV and $m_{h^0} = 65$ GeV, and extracts values for the oblique parameters. He finds $T_{\text{new}} = 0.70 \pm 0.21$, which in our model must arise from the contribution of the fourth generation fermions and the third and fourth generation squarks. (The contributions from other supersymmetric particles are negligible.) Using Eq. (5.1), we find that the fourth generation fermions yield a contribution of 0.25 to T_{new} . The contributions of the third and fourth generation squarks depend sensitively on the squark parameters as noted above; a range of parameters can be found that yields a total squark contribution to T_{new} between 0.3 and 0.4. Due to the infrared fixed point relation, the fourth generation mixing mass parameters $A_{t'} \simeq A_{b'} \simeq -155$ GeV. As an example, we find that if

$$\begin{aligned} m_Q \simeq m_U &\simeq 275 \text{ GeV}; & \mu &\simeq -460 \text{ GeV} \\ m_{Q'} \simeq m_{U'} &\simeq 250 \text{ GeV} & A_t &\simeq 695 \text{ GeV}, \end{aligned} \tag{5.2}$$

then the contribution from the third and fourth generation squarks to the T parameter is approximately equal to 0.4. Including the rest of the supersymmetric spectrum, we find $T_{\text{new}} \simeq 0.7$, which reproduces the central fitted value of T_{new} quoted above. As for S_{new} , its value is still subject to large errors: $S_{\text{new}} \simeq -0.16 \pm 0.21$ in the analysis of Ref. [41]. Summing the contributions from the fourth generation fermions and light supersymmetric particles yields a small positive contribution to S_{new} . For example, typical model parameter choices yield a value of $S_{\text{new}} \simeq 0.2$, which lies within the 95% confidence level bound deduced from precision electroweak data.

The exchange of third generation quarks and squarks contributes significant radiative corrections to the $Z \rightarrow b\bar{b}$ vertex. The fourth generation effects are suppressed by mixing angles and are hence small. Due to the smaller top Yukawa coupling, the radiative corrections arising from the virtual exchange of third generation squarks are smaller than those of the MSSM for the same choice of chargino and stop masses. In addition, the large value of $|\mu|$ implies that the light chargino is nearly a pure gaugino. Thus, even though the chargino and stop masses are close to $\frac{1}{2}m_Z$, the effect of the chargino-stop loop has only a small effect on the rate $Z \rightarrow b\bar{b}$. Our model then predicts

$$R_b \simeq 0.2184, \quad (5.3)$$

which, under the assumptions specified in section 2, is within one standard deviation of the measured LEP value [Eq. (2.3)]. The improvement over the Standard Model prediction is mainly due to the fact that $m_t \simeq m_W$.

The value of R_b has also consequences for the value of $\alpha_s(M_Z)$ extracted at LEP. The value of R_ℓ , from which $\alpha_s(M_Z)$ at LEP is extracted, would be different from the Standard Model value, due to the modification of Γ_{had} induced through the larger value of Γ_b . In models where only the bottom-quark width of the Z -boson is modified by virtual corrections, a simple relation between $\alpha_s(M_Z)$ and R_b may be obtained. In the one-loop approximation, it is given by

$$\frac{\alpha_s(M_Z)}{\pi} = \left(\frac{1 - R_b}{1 - R_b^0} \right) \left(\frac{\alpha_s^0(M_Z)}{\pi} \right) - \frac{R_b - R_b^0}{1 - R_b^0}, \quad (5.4)$$

where $\alpha_s^0(M_Z) = 0.124 \pm 0.005$ is the value that would be obtained from the measured value of R_ℓ in the Standard Model (assuming $m_{h^0} \simeq 60$ GeV), and $R_b^0 = 0.2156$ is the Standard Model value for R_b . Using the value of R_b predicted by our model [Eq. (5.3)], we find $\alpha_s(M_Z) \simeq 0.112 \pm 0.005$, in good agreement with determinations of α_s from deep inelastic scattering data [42].

6 Flavor Changing Neutral Current Processes

The low value of the top quark mass can have far reaching consequences for flavor changing neutral current (FCNC) processes. We shall analyze the predictions for three particularly sensitive FCNC observables: the $b \rightarrow s\gamma$ decay rate, the $B_d - \bar{B}_d$ mixing and the CP-violation parameter ϵ_K [43]. In our four generation supersymmetric model, the FCNC amplitudes depend not only on the top quark mass but also on the full set of supersymmetric parameters. In particular, the contribution of light stops and light charginos to these amplitudes can be quite significant. Moreover, the predictions depend strongly on the assumptions about the flavor structure of the soft-supersymmetry-breaking parameters. As an indicative example, we shall assume that a super-GIM mechanism is active. In addition, we further assume that the mixing of the fourth generation with the first and second generations is negligibly small. We shall present the results for $M_{\tilde{\chi}^\pm} \simeq 65$ GeV (the amplitudes considered below are slightly reduced for larger values of the chargino mass). The other model parameters are fixed by the phenomenological considerations of the previous sections.

One of the most sensitive tests of the model is to check that its prediction for $b \rightarrow s\gamma$ is consistent with $1.0 \times 10^{-4} \lesssim \text{BR}(b \rightarrow s\gamma) \lesssim 4 \times 10^{-4}$, as required by the CLEO measurement [44]. In Figure 4 we plot the prediction for the branching ratio $\text{BR}(b \rightarrow s\gamma)$ in our model. It follows that, for values of the CP-odd Higgs mass $m_A \gtrsim 100$ GeV, the prediction of our model lives comfortably within the experimental bound.

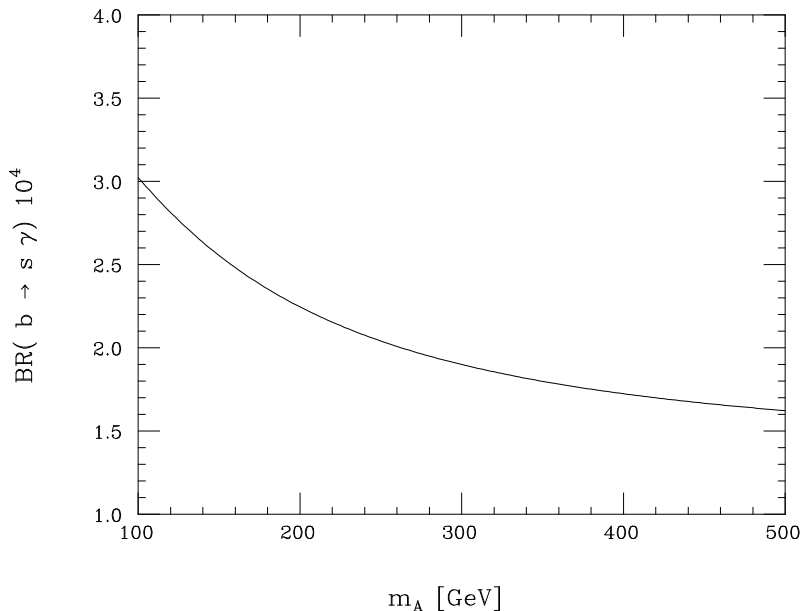


Figure 4. Branching ratio $\text{BR}(b \rightarrow s\gamma)$ as a function of the CP-odd mass m_A .

The bounds on B_d - \overline{B}_d mixing and ϵ_K can place a lower bound on the Standard Model top quark mass value. This bound can be explicitly obtained from the following two equations taken from Ref. [45]. From the measurement of ϵ_K , one deduces that:

$$\eta \left[(1 - \rho) A^2 \eta_2 P + P_c \right] A^2 B_K \simeq 0.223, \quad (6.1)$$

where η and ρ are parameters arising in the Wolfenstein parameterization of the CKM matrix (which are constrained by the relation $\sqrt{\rho^2 + \eta^2} = 0.355 \pm 0.090$), $A \equiv |V_{cb}|/|V_{us}|^2$ is found from data to be $A = 0.826 \pm 0.060$, $\eta_2 \simeq 0.57$ is a QCD correction factor, P is a reduced amplitude that arises in the computation of the W - t box diagram, $P_c \simeq 0.3$ is a small correction that arises from the box diagram mediated by the charmed quark, and $B_K = 0.8 \pm 0.2$ represents the uncertainty in the hadronic matrix element.

From the measurement of B_d - \overline{B}_d mixing, one deduces that:

$$\left| \frac{V_{td}}{V_{us} V_{cb}} \right|^2 \left(\frac{0.75}{x_d} \right) \left(\frac{F_B \sqrt{B_B}}{200 \text{ MeV}} \right)^2 \left| \frac{V_{cb}}{0.040} \right|^2 \left(\frac{\tau_B}{1.6 \text{ ps}} \right)^2 P \simeq 2.089, \quad (6.2)$$

where $F_B \sqrt{B_B} = 200 \pm 40$ MeV, $|V_{cb}| = 0.040 \pm 0.003$, $\tau_B = 1.6 \pm 0.1$ ps, and $x_d \equiv \Delta M_B / \Gamma_B = 0.75 \pm 0.06$.

In the Standard (non-supersymmetric three-generation) Model, the reduced amplitude P depends on the top quark mass according to the following approximate formula:

$$P_{\text{SM}} \simeq 0.79 \left(\frac{M_t}{m_W} \right)^{1.52}, \quad (6.3)$$

where $M_t \equiv m_t(m_t)$ is the $\overline{\text{MS}}$ running top quark mass evaluated at the pole mass. For example, for a running mass $M_t = 170$ GeV, $P \simeq 2.465$. Scanning over the allowed values of all free parameters at the one sigma level we find

$$0.8 \lesssim P \lesssim 10.5. \quad (6.4)$$

The lower bound on P is relevant for our discussion. In particular, the lower bound on P translates into a lower bound on the top quark mass given by $M_t \gtrsim 80$ GeV.

We can now apply this analysis to any model for which an effective GIM mechanism is active. In particular, let P_4 be the value of P obtained in our four-generation low-energy supersymmetric model. We find that P_4 is enhanced with respect to P_{SM} for the same top quark mass, primarily due to the effects of light supersymmetric particle exchange. Figure 5 shows the ratio P_4/P_{SM} as a function of the CP-odd Higgs mass. We see that the enhancement factor is approximately 1.35, with a dependence on m_A an indication of the size of the charged Higgs exchange. Hence, the reduced amplitude in our model is $P_4 \simeq 1.01$; which is within the one sigma bound quoted above. Note that this value strongly depends on our assumption about the presence of an effective GIM mechanism. If the model contains extra flavor violation parameters, the prediction for P_4 can be significantly modified. Furthermore, if new CP-violation phases are present in the theory, the above bounds on P_4 [Eq. (6.4)] can be further relaxed.

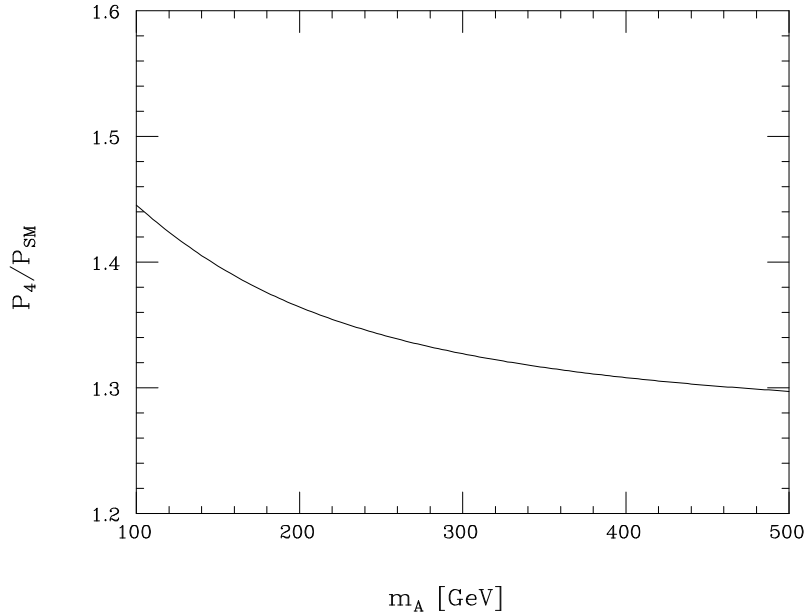


Figure 5. Ratio of the $B_d\text{--}\overline{B}_d$ reduced amplitude P_4 in the four generation model to the corresponding amplitude P_{SM} in the Standard Model, for $m_t = 80$ GeV as a function of the CP-odd Higgs mass m_A .

7 Discussion and Conclusions

A low energy supersymmetric model with four generations has been presented with $m_t \simeq m_W$, which satisfies all present day phenomenological constraints. Typical values of the model spectrum are shown in Table 1. The fourth generation quark t' is identified as the state discovered by the CDF and D0 Collaborations, while the light top quark decays via $t \rightarrow \tilde{t}\tilde{\chi}_1^0$, thereby escaping previous searches at hadron colliders. The oblique radiative corrections inferred from precision electroweak measurements at LEP are reproduced by contributions from the third and fourth generation quarks and squarks. The most theoretically troubling feature of the model is the large mixing among the third generation squarks that is necessary to ensure a light top squark and provide a viable prediction for the electroweak ρ -parameter.

There are several remarkable properties of the present model in contrast to the MSSM which should be stressed:

1. In the MSSM, any physics leading to larger values of R_b also contributes to non-standard top quark decays [12]. Indeed, the potentially viable scenarios invoke either light charginos² and light stops for small values of $\tan\beta$ and/or light charged Higgs bosons for large values of $\tan\beta$. Light charginos are usually associated with light neutralinos; hence, the decay mode $t \rightarrow \tilde{t}\tilde{\chi}_1^0$ is present, which can account for a significant fraction of the top quark branching ratio. As a result, in such models, the typical branching ratio of non-Standard Model top quark decays is of order 30%.

²Based on the absence of light charginos in the most recent LEP run at $\sqrt{s} = 136$ GeV, Ref. [13] quotes an absolute upper limit of $R_b < 0.2174$ for the three-generation MSSM in the case of small $\tan\beta$.

Table 1. Example of Model Spectrum and Parameters ($\tan \beta = 1.6$)

Particle	Mass (GeV)	Particle	Mass (GeV)
t	80	$\tilde{\chi}_1^0$	25
b'	105	$\tilde{\chi}_1^+$	65
t'	175	$\tilde{\chi}_2^0$	66
τ'	50	\tilde{g}	190
$N_{\tau'}$	80	\tilde{b}	264
h^0	67	\tilde{t}	50
A^0	300	\tilde{b}'	116
H^\pm	310	\tilde{t}'	263
Parameter	Value (GeV)	Parameter	Value (GeV)
μ	-460	$M_{\tilde{Q}, \tilde{U}, \tilde{D}}$	275
M_1	23	$M_{\tilde{Q}', \tilde{U}', \tilde{D}'}$	250
M_2	54	$A_{t,b}$	695
M_3	170	$A_{t',b'}$	-155

This is in slight disagreement with the Tevatron top decays observations, although the experimental errors are still large enough to permit a non-negligible reduction of $\text{BR}(t \rightarrow bW^+)$ from 1. In our model, the decay of $t' \rightarrow \tilde{t}\tilde{\chi}^0$ is a flavor violation process, which only occurs at the one loop level, and hence is naturally suppressed relative to the decay $t' \rightarrow bW^\pm$.

2. The large value of the top quark mass in the MSSM tends to lead to FCNC processes (which are particularly striking in the leptonic sector) induced by physics at the grand unification scale [46]. In our case, these effects are reduced due to the lower value of the top quark Yukawa coupling at M_{GUT} and the expected small mixing between the fourth and the first two generations.

3. The model presented in this paper exhibits the unification of the four fourth generation fermion-Higgs Yukawa couplings at a common high energy scale. In the MSSM, the unification of the bottom and top quark Yukawa couplings can only be obtained for very large values of $\tan \beta$, which usually implies a large fine tuning of the Planck scale parameters [47]. In contrast, $\tan \beta \sim \mathcal{O}(1)$ in our model, which is a more natural value for a model of bottom-top Yukawa coupling unification (and is certainly less fine-tuned than the large $\tan \beta$ model), especially given the proximity of the Yukawa couplings to their infrared fixed point values.

In a complementary work, Gunion, McKay and Pois [48] constructed four-generation models in the framework of minimal low-energy supergravity. The state discovered by the CDF and D0 Collaborations near 175 GeV in mass was taken to be the top quark. Thus, to keep the fourth generation quark-Higgs Yukawa couplings perturbative up to

the GUT scale, one must hide the b' and t' in a mass region below $m_t \simeq 175$ GeV. In this alternative approach, the R_b anomaly would have to be explained by assuming an appropriately light super-partner spectrum (as in the case of the three-generation MSSM). Gunion *et al.* also performed a complete renormalization group analysis of the soft-supersymmetry breaking terms in the four-generation models. In particular, they demonstrated that our model cannot be obtained from universal boundary conditions for the soft-supersymmetry-breaking parameters at M_{GUT} . This result was due primarily to the large values of $A_t(M_{\text{GUT}})$ required to achieve a light top squark. Although helpful in suppressing flavor changing processes, the universality of scalar masses is not a necessary condition for obtaining an acceptable phenomenology. Even in the case of the three-generation MSSM, the violation of the universal boundary conditions seems to be a requirement for obtaining values of R_b closer to the present experimental values [7,11,49].

We have demonstrated that present day experimental data place severe constraints on the four-generation model parameters. Moreover, the model possesses a rich spectrum of new particles that will be accessible to LEP-2 and the Tevatron. In particular, eight new particles of this model could be discovered at LEP-2: the t -quark, the fourth generation leptons (τ' and $N_{\tau'}$), the light Higgs boson (h^0), and four supersymmetric particles ($\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, $\tilde{\chi}_1^\pm$, and \tilde{t}). New bounds coming from the initial run of LEP-2 at $\sqrt{s} = 136$ GeV have constrained the model further. For instance, non-observation of the τ' requires that it must be almost degenerate in mass with the fourth generation neutrino. Moreover, the non-observation of the lightest chargino pushes its mass to its upper allowed range, $M_{\tilde{\chi}_\pm} \simeq 65\text{--}70$ GeV. One can imagine scenarios where some of the properties of the eight particles listed above are altered to avoid conflict with future collider limits.³ However, it is certain that our model is ruled out unless a light top squark is discovered in LEP-2 data expected to be collected in 1996, or perhaps in the Run Ib data from the Tevatron after the complete data samples have been analyzed. Of course, if such a state is discovered, it could be consistent with the three-generation MSSM. In this case, the light top quark in our approach (or a fourth generation quark doublet with mass below that of the top quark in the approach of Gunion *et al.*) must be found to confirm the existence of a new generation. Alternatively, the exclusion of these models would raise ones confidence that the number of families in the effective

³As an example, if gaugino masses are not unified then the mass of $\tilde{\chi}_1^\pm$ is independent of the mass of the lightest neutralino if both states are dominantly gaugino-like (*e.g.*, assuming $|\mu|$ is large compared to the gaugino mass parameters). Then, by choosing the $\tilde{\chi}_1^\pm$ mass above 100 GeV, one can tolerate a substantially lighter \tilde{b}' than shown in Table 1, while still kinematically forbidding the two-body decay $t' \rightarrow \tilde{b}'\tilde{\chi}_1^+$. (A lighter \tilde{b}' can be obtained by adjusting μ to generate larger mixing among the fourth generation squarks.) Three “benefits” ensue: (i) a lighter \tilde{b}' can generate a larger fourth generation squark contribution to $\Delta\rho$ and increase the radiatively corrected value of m_{h^0} , thus allowing for less extreme mixing parameters in the third generation squark sector; (ii) the b' quark would be more difficult to detect at the Tevatron, since in this case $b' \rightarrow \tilde{b}'\tilde{\chi}_1^0$ followed by $\tilde{b}' \rightarrow b\tilde{\chi}_1^0$ would be the dominant decay chain; and (iii) a heavier chargino would be kinematically inaccessible to LEP-2.

low-energy supersymmetric theory below M_{GUT} is indeed three.

Acknowledgements

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Appendix A Squark contributions to the ρ -parameter

This appendix contains the formulae for the squark contributions to the ρ -parameter. Only the contribution of the third generation squarks will be written out explicitly below. The contributions of the other squark generations can be trivially obtained by employing the appropriate squark masses and mixing angles (to a good approximation, squark mixing is appreciable only in the third and fourth generations). The formulae below can also be applied to the slepton contributions by setting the color factor ($N_c = 3$ for squarks) equal to unity.

We define the squark mixing angles as follows. Let \tilde{q}_1 and \tilde{q}_2 be the quark mass eigenstates, with $M_{\tilde{q}_1} < M_{\tilde{q}_2}$. (In this paper, we have denoted the light squark eigenstate \tilde{q}_1 by \tilde{q} , since the heavier squark eigenstate does not play an important role in our considerations.) Then, in terms of the interaction eigenstates \tilde{q}_L and \tilde{q}_R ,

$$\begin{aligned}\tilde{q}_1 &= -\tilde{q}_L \sin \theta_q + \tilde{q}_R \cos \theta_q, \\ \tilde{q}_2 &= \tilde{q}_L \cos \theta_q + \tilde{q}_R \sin \theta_q.\end{aligned}\tag{A.1}$$

For example, after diagonalizing the top-squark mass-squared matrix [Eq. (4.1)], one obtains

$$\tan 2\theta_t = \frac{2m_t(A_t - \mu \cot \beta)}{M_Q^2 - M_U^2 + (c_L - c_R)m_Z^2}.\tag{A.2}$$

The contribution of the third generation squarks to the ρ -parameter is then given by:

$$\begin{aligned}\Delta\rho(\tilde{t}, \tilde{b}) &= \frac{g^2 N_c}{32\pi^2 m_W^2} \left[\sin^2 \theta_t \sin^2 \theta_b F(m_{\tilde{t}_1}^2, m_{\tilde{b}_1}^2) + \sin^2 \theta_t \cos^2 \theta_b F(m_{\tilde{t}_1}^2, m_{\tilde{b}_2}^2) \right. \\ &\quad + \cos^2 \theta_t \sin^2 \theta_b F(m_{\tilde{t}_2}^2, m_{\tilde{b}_1}^2) + \cos^2 \theta_t \cos^2 \theta_b F(m_{\tilde{t}_2}^2, m_{\tilde{b}_2}^2) \\ &\quad \left. - \sin^2 \theta_t \cos^2 \theta_b F(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) - \sin^2 \theta_b \cos^2 \theta_t F(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2) \right]\end{aligned}\tag{A.3}$$

where the function F is defined by

$$F(m_1^2, m_2^2) \equiv \frac{1}{2}(m_1^2 + m_2^2) - \frac{m_1^2 m_2^2}{m_1^2 - m_2^2} \ln \left(\frac{m_1^2}{m_2^2} \right). \quad (\text{A.4})$$

Note that in the supersymmetric limit ($M_{\tilde{t}_1} = M_{\tilde{t}_2} = m_t$ and $M_{\tilde{b}_1} = M_{\tilde{b}_2} = m_b$), one finds $\Delta\rho(\tilde{t}, \tilde{b}) = \Delta\rho(t, b)$ [see Eq. (5.1)]. In contrast, in the limit where $M_Q^2 \gg m_Z^2$, m_t^2 , the squark contribution to the ρ -parameter decouples:

$$\Delta\rho(\tilde{t}, \tilde{b}) \simeq \frac{g^2 N_c m_t^4 K}{32\pi^2 m_W^2 M_Q^2}, \quad (\text{A.5})$$

where K approaches a constant in the limit of large M_Q^2 . One can evaluate K by using the following properties of $F(m_1^2, m_2^2)$ in the limit of $|m_1^2 - m_2^2| \ll m_1^2, m_2^2$

$$\begin{aligned} F(m_1^2, m_2^2) &\simeq \frac{(m_1^2 - m_2^2)^2}{6m_2^2}, \\ F(m_1^2, m_3^2) - F(m_2^2, m_3^2) &\simeq (m_1^2 - m_2^2) \left[\frac{1}{2} + \frac{m_3^2}{m_3^2 - m_2^2} + \frac{m_3^4}{(m_3^2 - m_2^2)^2} \ln \left(\frac{m_2^2}{m_3^2} \right) \right]. \end{aligned} \quad (\text{A.6})$$

In this paper, it is the important third and fourth generation squark mixing effects that lead to a significant supersymmetric contribution to the ρ -parameter.

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